Solutions To Project 6

(1)

$$y = 5 - 2x^2 + x^4$$
$$y' = -4x + 4x^3$$

(2)

$$y = 4x^3 + 2 + \frac{1}{x}$$
$$y' = 12x^2 - \frac{1}{x^2}$$

(3)

$$y = 2\sqrt{x} - \frac{3}{\sqrt[3]{x}} = 2x^{\frac{1}{2}} - 3x^{-\frac{1}{3}}$$
$$y' = x^{-\frac{1}{2}} + x^{-\frac{4}{3}} = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x^4}}$$

(4)

$$y = \frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$$
$$y' = -\frac{2}{3}x^{-\frac{5}{3}} = -\frac{2}{3\sqrt[3]{x^5}}$$

(5)

$$y = (x+1)(x^3+3)$$
$$y' = 1.(x^3+3) + (x+1)(3x^2)$$
$$y' = x^3 + 3 + 3x^3 + 3x^2$$
$$y' = 4x^3 + 3x^2 + 3$$

Note: Also can use foil rule to express y as a polynomial and then differentiate.

(6)

$$y=\frac{3-x}{x^2-3}$$

$$y'=\frac{(x^2-3)(-1)-(3-x)(2x)}{(x^2-3)^2}$$

$$y'=\frac{-x^2+3-6x+2x^2}{(x^2-3)^2}$$

$$y'=\frac{x^2-6x+3}{(x^2-3)^2}$$

(7)

$$y=xe^{-x^2}$$

$$y'=1e^{-x^2}+xe^{-x^2}(-2x)$$

$$y'=e^{-x^2}(1-2x^2)$$

(8)

$$y=\ln(1+e^x)$$

$$y'=\frac{e^x}{1+e^x}$$

(9)

$$y=\sqrt{\ln(x)}=(\ln(x))^{\frac{1}{2}}$$

$$y'=\frac{1}{2}(\ln(x))^{-\frac{1}{2}}.\frac{1}{x}=\frac{1}{2\left(\sqrt{\ln(x)}\right)x}$$

(10)

$$y=\frac{e^{x^2}}{e^x}=e^{x^2-x}$$

$$y'=e^{x^2-x}(2x-1)=\frac{e^{x^2}(2x-1)}{e^x}$$

(11)

$$y=\ln(3)\ln(x)$$

$$y'=\ln(3).\frac{1}{x}=\frac{\ln(3)}{x}$$

(12)

$$y = 3x^5 + 7x^3 - 4x^2 + 12$$
$$y' = 15x^4 + 21x^2 - 8x$$

(13)

$$y = x^{\sqrt{2}} \ln(x)$$
$$y' = \sqrt{2}x^{\sqrt{2}-1} \ln(x) + x^{\sqrt{2}} . \frac{1}{x}$$
$$y' = 1.414x^{.414} \ln(x) + x^{\sqrt{2}-1}$$
$$y' = 1.414x^{.414} \ln(x) + x^{.414}$$
$$y' = x^{.414}(1.414 \ln(x) + 1)$$

(14)

$$y = \frac{\ln(x)}{x}$$
$$y' = \frac{x.\frac{1}{x} - \ln(x).1}{x^2} = \frac{1 - \ln(x)}{x^2}$$

(15)

$$y = (1 + \sqrt{x})(3 - 2x)$$
$$y' = \frac{1}{2\sqrt{x}}(3 - 2x) + (1 + \sqrt{x})(-2)$$
$$y' = \frac{3}{2\sqrt{x}} - \sqrt{x} - 2 - 2\sqrt{x}$$
$$y' = \frac{3}{2\sqrt{x}} - 3\sqrt{x} - 2$$

Note: Also can use foil rule and then differentiate.

(16)

$$y = \frac{8}{x+1} = 8(x+1)^{-1}$$
$$y' = -8(x+1)^{-2}(1) = -\frac{8}{(x+1)^2}$$

(17)

$$y=\frac{5x^2-7x+4}{x}=5x-7+\frac{4}{x}$$

$$y'=5-\frac{4}{x^2}$$

(18)

$$y=\left(x^2-5x+4\right)^3$$

$$y'=3\left(x^2-5x+4\right)^2(2x-5)$$

(19)

$$y=\sqrt{1+e^x}=\left(1+e^x\right)^{\frac{1}{2}}$$

$$y'=\frac{1}{2}\left(1+e^x\right)^{-\frac{1}{2}}.e^x$$

$$y'=\frac{e^x}{2\sqrt{1+e^x}}$$

(20)

$$y=\frac{1}{\sqrt[3]{x-\ln(x)}}=\left(x-\ln(x)\right)^3$$

$$y'=-\frac{1}{3}\left(x-\ln(x)\right)^{-\frac{4}{3}}\left(1-\frac{1}{x}\right)$$

(21) And (22)

$$P=\frac{5}{1+R}+\frac{5}{(1+R)^2}+\frac{5}{(1+R)^3}+\frac{105}{(1+R)^4}$$

$$\frac{dP}{dR}=\frac{-5}{(1+R)^2}-\frac{10}{(1+R)^3}-\frac{15}{(1+R)^4}-\frac{410}{(1+R)^5}$$

$$\frac{d^2P}{dR^2}=\frac{10}{(1+R)^3}+\frac{30}{(1+R)^4}+\frac{60}{(1+R)^5}+\frac{2100}{(1+R)^6}$$

(23)

$$s = f(t) = \frac{1}{3}t^3 - 2t^2 + 3t ; 0 \le t$$

(a)

$$V_t = f'(t) = t^2 - 4t + 3$$
$$V_t = 0 \quad \text{when} \quad (t-3)(t-1) = 0$$
$$t = 3 \quad \text{or} \quad t = 1$$
$$\text{Now} \quad f(1) = 1.333 \quad \text{and} \quad f(3) = 0$$

(b)

$$a_t = f''(t) = 2t - 4$$
$$a_t = 0 \quad \text{when} \quad 2(t-2) = 0$$
$$t = 2$$
$$\text{Now} \quad f(2) = .667$$

(24)

$$\frac{dZ}{dX}(2) = (fog)'(2) = f'(g(2)).g'(2)$$
$$= f'(3).4$$
$$= 5 \times 4 = 20$$

(25)

(a)	F	$(X_t)' = a_t$
(b)	F	Product Rule
(c)	T	
(d)	T	
(e)	F	$\left(X^{\frac{m}{n}}\right)' = \frac{m}{n}X^{\frac{m}{n}-1}$
(f)	T	
(g)	F	Quotient Rule