Solutions To Project 4

(1a)

$$(4,-2),(-1,7)$$
$$D=\sqrt{(-1-4)^2+(7-(-2))^2}$$
$$D=\sqrt{(-5)^2+(9)^2}=\sqrt{74}=8.60$$

(1b)

$$(2,5),(3,1)$$
$$D=\sqrt{(3-2)^2+(1-5)^2}$$
$$D=\sqrt{(1)^2+(-4)^2}=\sqrt{17}=4.12$$

(1c)

$$(0,0),(3,4)$$
$$D=\sqrt{(3-0)^2+(4-0)^2}$$
$$D=\sqrt{(3)^2+(4)^2}=\sqrt{25}=5$$

(1d)

$$(-4,3),(-4,-5)$$
$$D=\sqrt{(-4-(-4))^2+(-5-3)^2}$$
$$D=\sqrt{(0)^2+(-8)^2}=\sqrt{64}=8$$

Note: $D=8$ immediately from observing points on coordinate system.

(2)

$$l_1 : y=-\frac{1}{4}x+2$$
$$l_2 : x=-3$$
$$l_3 : y=-2$$
$$l_4 : y=\frac{3}{2}x$$

(3a)

$$\frac{3}{2}x = -\frac{1}{4}x + 2$$

$$\frac{7}{4}x = 2$$

$$x = \frac{4}{7}(2) = \frac{8}{7} \Rightarrow y = \frac{3}{2}\left(\frac{8}{7}\right) = \frac{12}{7}$$

The point is $\left(\frac{8}{7}, \frac{12}{7}\right)$

(3b)

$$-2 = \frac{3}{2}x$$

$$-4 = 3x$$

$$x = -\frac{4}{3}$$

$$y = -2$$

The point is $\left(-\frac{4}{3}, -2\right)$

(3c)

The point is $(-3, -2)$

(4)

$$(-1,3), (4,-9)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-9-3}{4-(-1)} = \frac{-12}{5} = -2.4$$

$$y - 3 = -2.4(x - (-1))$$

$$y - 3 = -2.4x - 2.4$$

$$y = -2.4x + .6$$

(5)

$$4x - 5y = 10$$

$$-5y = -4x + 10$$

$$y = \frac{4}{5}x - 2$$

$$m = .8$$

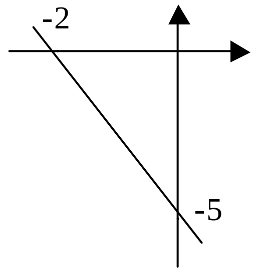
$$(2,-1)$$

$$y - (-1) = .8(x - 2)$$

$$y + 1 = .8x - 1.6$$

$$y = .8x - 2.6$$

(6)



$$m = \frac{\Delta y}{\Delta x} = -\frac{5}{2} = -2.5$$
$$y - 0 = -2.5(x - (-2))$$
$$y = -2.5x - 5$$

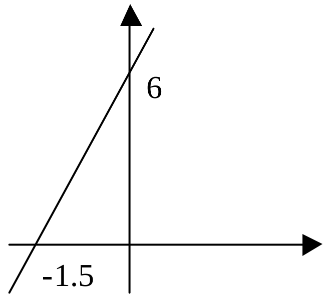
(7)

x	-5	a	5	10
y	b	3	6	8

$$m = \frac{\Delta y}{\Delta x} = \frac{8-6}{10-5}$$
$$m = \frac{2}{5} = .4$$
so,
$$\frac{6-3}{5-a} = \frac{3}{5-a} = .4 \Rightarrow 3 = 2 - .4a$$
$$.4a = -1$$
$$a = \frac{-1}{.4} = -2.5$$
and
$$\frac{6-b}{5-(-5)} = \frac{6-b}{10} = .4 \Rightarrow 6 - b = 4$$
$$b = 2$$

(8)

$$-8x + 2y = 12$$
x axis : y=0 so $-8x = 12 \Rightarrow x = -1.5$
y axis : x=0 so $2y = 12 \Rightarrow y = 6$
$$m = \frac{rise}{run} = \frac{6}{1.5}$$
$$m = 4$$



(9)

$$f(x) = \sqrt{25 - x^2}$$

$$f(-4) = \sqrt{25 - (-4)^2} = \sqrt{25 - 16} = 3$$

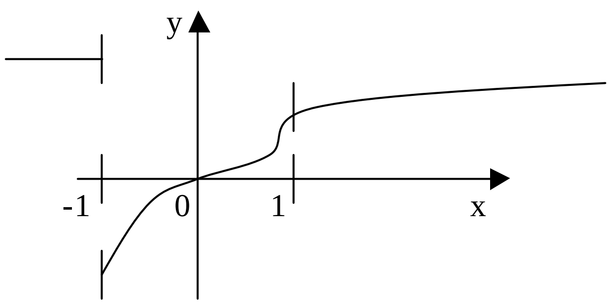
$$f(6) = \sqrt{25 - (6)^2} = \sqrt{-11}, \text{ Not defined}$$

domain is $\{x \mid -5 \le x \le 5\}$

Note: $0 \le 25 - x^2 \Leftrightarrow x^2 \le 25 \Leftrightarrow -5 \le x \le 5$

(10)

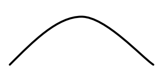
$$y = \begin{cases} 2 \text{ if } x \le -1 \\ x^3 \text{ if } -1 < x \le 1 \\ \sqrt{x} \text{ if } 1 \le x \end{cases}$$



(11)

$$y = 6 - x - x^2 = f(x)$$

(a) a frowning parabola



(b) $c = 6$

(c)

maximum $-\frac{b}{2a} = \frac{1}{2(-1)} = -\frac{1}{2}$

$c - \frac{b^2}{4a} = 6 - \frac{1}{-4} = 6\frac{1}{4}$

The point is (-.5, 6.25)

(d)

$y = 0$ so

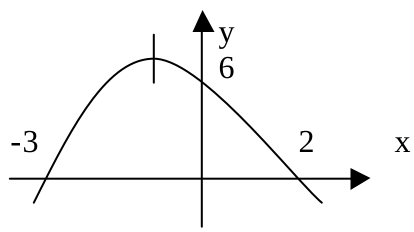
$0 = 6 - x - x^2$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3, 2$

(e)



(12)

$3x^2 - 6x - 13 = x^2 - 7x + 2$

$2x^2 + x - 15 = 0$

$(2x-5)(x+3) = 0$

$x = -3, \frac{5}{2} \Rightarrow (-3,32)$

$(2.5,-9.25)$

(13)

$x^2 - x - 9 = 2x + 1$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = -2,5 \Rightarrow (-2,-3)$

$(5,11)$

(14)

$$f(x)=\frac{2}{x-1}-4$$

$X - axis: y=0$

$$so \quad 0=\frac{2}{x-1}-4$$

$$4=\frac{2}{x-1}$$

$$4x-4=2$$

$$4x=6 \Rightarrow x=1.5$$

$Y - axis: x=0$

$$so \quad y=\frac{2}{0-1}-4$$

$$y=-2-4$$

$$y=-6$$

(15)

$\Delta x=1$

Δy: ratios constant at 2.5

So $y=12(2.5)^x$

(16)

$\Delta x = 4 \Rightarrow \frac{1}{k}=4$ i.e. $k=\frac{1}{4}$

Δy: ratios constant at .75

So $y=800\times(.75)^{\frac{1}{4}x}$

(17)

$A_0=8 \qquad B=3$

(a) $A=8\times(3)^t$

(b)

$t=13 \Rightarrow A=8\times(3)^{13}$

$A=12{,}754{,}584$

Weight is 12,754.58 lbs
Value is $63,772.92

(18)

$$A = Pe^{RT}$$

(a)

$$3P = Pe^{R5}$$

$$3 = e^{5R}$$

$$\ln(3) = 5R \Rightarrow R = \frac{\ln(3)}{5} = .22$$

(b)

$$3P = Pe^{RT}$$

$$3 = e^{RT}$$

$$\ln(3) = RT \Rightarrow R = \frac{\ln(3)}{T} = \frac{1.1}{T}$$

(c)

$$NP = Pe^{RT}$$

$$N = e^{RT}$$

$$\ln(N) = RT \Rightarrow R = \frac{\ln(N)}{T}$$

(19a)

$$\left.\begin{array}{l}\Delta T = 2 \\ \Delta A = 3\end{array}\right\}$$ A Vs T is linear with slope m=1.5

So

$$A - 15 = 1.5(T - 1)$$

$$A - 15 = 1.5T - 1.5$$

$$A = 1.5T + 13.5$$

(19b)

if T=0 then A=13.5 so initial investment is $13,500

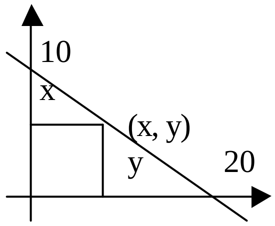
(19c)

$$A = P(1 + RT) = P + PRT$$

so $PR = 1.5 = \$1500$

and $R = \frac{1500}{13{,}500} = .11$

(20)



$$A = xy$$

$$y = \left(-\frac{1}{2}x + 10\right)$$

$$A = x\left(-\frac{1}{2}x + 10\right)$$

$$A = -\frac{1}{2}x^2 + 10x$$

A has a maximum value at $-\frac{b}{2a}$

$= -\frac{10}{2\left(-\frac{1}{2}\right)} = 10$ i.e. $X_{\max} = 10$

$Y_{\max} = 5$ and $A_{\max} = 50$

(21a)

$$P = \frac{5}{1.03} + \frac{5}{(1.03)^2} + \frac{5}{(1.03)^3} + \frac{105}{(1.03)^4}$$

$$P_{.03} = 107.43$$

(21b)

$$P_{.05} = 100$$

(21c)

$$P_{.07} = 93.23$$

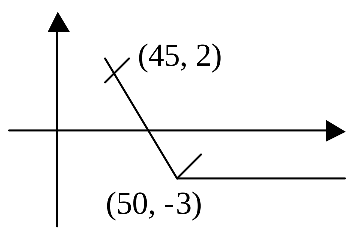
(21d)

As R increases P decreases

(22)

$$f(x) = x - x^2$$
$$f(x+h) = (x+h) - (x+h)^2$$
$$f(x+h) = x + h - (x^2 + 2xh + h^2)$$
$$f(x+h) = x + h - x^2 - 2xh - h^2$$
$$\frac{f(x+h) - f(x)}{h} = \frac{h - 2xh - h^2}{h}$$
$$= \frac{h(1 - 2x - h)}{h}$$
$$= 1 - 2x - h$$

(23)



$$\frac{\Delta y}{\Delta x} = \frac{-3-2}{50-45} = \frac{-5}{5} = -1$$
$$y - 2 = -1(x - 45)$$
$$y - 2 = -x + 45$$
$$y = -x + 47$$

(a)

$$y = \begin{cases} -x + 47 & \text{if} \quad x \le 50 \\ -3 & \text{if} \quad 50 \le x \end{cases}$$

(b)

$$y = 0 : 0 = -x + 47 \Rightarrow x = 47$$

(c)

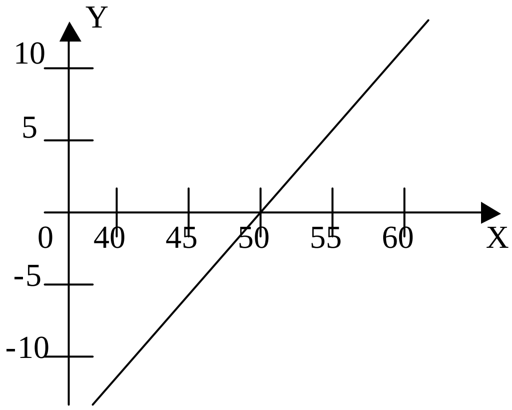
If $x = 40$ then $y = -40 + 47 = 7$

If $x = 55$ then $y = -3$

(24a)

X	40	45	50	55	60
Y	-10	-5	0	5	10

(24b)



This graph is a line with positive slope.

(24c)

$$m = \frac{\Delta y}{\Delta x} = \frac{5-0}{55-50} = \frac{5}{5} = 1$$

$$y - 0 = 1(x - 50)$$

$$y = x - 50\,;\, 0 \le x$$

This is a <u>linear</u> function!

(25a)

$$y = x + \frac{1}{x}$$

x intercept $: y = 0$

$$0 = x + \frac{1}{x}$$

$$0 = \frac{x^2 + 1}{x}\,;\, x \neq 0$$

$$x^2 = -1$$

no x intercepts .

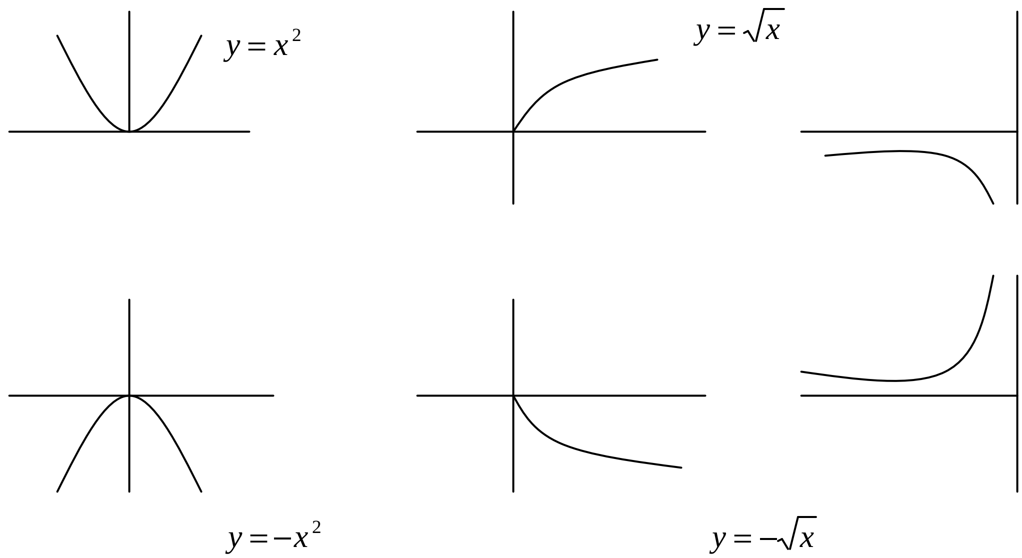
y intercepts $: x = 0$ But $x \neq 0$

because $\frac{1}{x}$ not defined for x $= 0$

no y intercepts .

(25b)

The rule is valid.



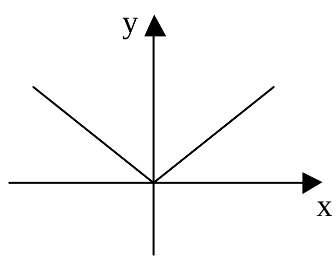
(25c)

$f(x) = 1 \Rightarrow f(x+h) = 1$

(25d)

yes. The NAV can be tracked as the dependent variable Vs the independent variable of

(25e)



$f(-7) = -(-7)$

$f(-7) = 7$

$f(3) = 3$

(25f)

$$z = 2y^2 + 1 = f(y)$$

$$y = 3x - 5 = g(x)$$

$$(fog)(x) = f(g(x))$$

$$= f(3x - 5)$$

$$= 2(3x - 5)^2 + 1$$

$$= 2(9x^2 - 30x + 25) + 1$$

$$= 18x^2 - 60x + 50 + 1$$

$$(fog)(x) = 18x^2 - 60x + 51$$