

Solutions To Project 3

(1)

$$25,000 = 1000(1.25)^T$$

$$25 = 1.25^T$$

$$\ln(25) = T \ln(1.25)$$

$$3.219 = T(.223)$$

$$T = 14.435$$

(2)

$$\sqrt[4]{X^3} = 16$$

$$X^3 = 16^4$$

$$X^3 = 65,536$$

$$X = 40.317$$

(3)

$$8000 = 10000(1+R)^{5.7}$$

$$.8 = (1+R)^{5.7}$$

$$(.8)^{1/5.7} = (1+R)$$

$$.9616 = 1+R$$

$$R = -.0384$$

(4)

$$A = P \left[\frac{(1+R)^n - 1}{R} \right]; n$$

$$\frac{AR}{P} = (1+R)^n - 1$$

$$\frac{AR}{P} + 1 = (1+R)^n$$

$$\ln\left(\frac{AR}{P} + 1\right) = n \ln(1+R)$$

$$n = \frac{\ln\left(\frac{AR}{P} + 1\right)}{\ln(1+R)}$$

(5)

$$\sqrt{1+X} = 8$$

$$1+X = 64$$

$$X = 63$$

(6)

$$\begin{aligned}2^{(x^2-7x+8)} &= 100 \\(x^2 - 7x + 8)\ln(2) &= \ln(100) \\x^2 - 7x + 8 &= 6.644 \\x^2 - 7x + 1.356 &= 0 \\ \frac{-(-7) \pm \sqrt{49 - 4(1)(1.356)}}{2} \\ \frac{7 \pm \sqrt{43.576}}{2} \\ x &= 6.8 \\ \text{or} \\ x &= .1994\end{aligned}$$

(7)

$$\begin{aligned}10,000(1.2)^T &= 25,000(1.1)^T \\ T \ln(1.2) &= \ln(2.5) + T \ln(1.1) \\ .1823T &= .9163 + .09531T \\ .08699T &= .9163 \\ T &= 10.533\end{aligned}$$

(8)

$$\begin{aligned}\log(X^2) - \log(X) &= 4 \\ \log\left(\frac{X^2}{X}\right) &= 4 \\ 10^{\log(X)} &= 10^4 \\ X &= 10^4 = 10,000\end{aligned}$$

(9)

$$\begin{aligned}24,000e^{.05(T-3)} &= 15000(1.08)^T \\ e^{.05(T-3)} &= .625(1.08)^T \\ .05T - .15 &= \ln(.625) + T \ln(1.08) \\ .05T - .15 &= -.47 + .077T \\ .027T &= .32 \\ T &= 11.85\end{aligned}$$

(10)

$$F = Se^{(R-Y)T}; Y$$

$$\frac{F}{S} = e^{(R-Y)T}$$

$$\ln\left(\frac{F}{S}\right) = (R-Y)T$$

$$\frac{\ln\left(\frac{F}{S}\right)}{T} = R - Y$$

$$Y = R - \frac{\ln\left(\frac{F}{S}\right)}{T}$$

(11)

$$P = \frac{100}{(1+R)^n}; R$$

$$P(1+R)^n = 100$$

$$(1+R)^n = \frac{100}{P}$$

$$1+R = \left(\frac{100}{P}\right)^{\frac{1}{n}}$$

$$R = \left(\frac{100}{P}\right)^{\frac{1}{n}} - 1$$

(12)

$$.5X^6 - 1000 = 0$$

$$.5X^6 = 1000$$

$$X^6 = 2000$$

$$X = 3.5496$$

(13)

$$1000e^{.1t} = -20,000$$

$$\ln(e^{.1t}) = (-20)\ln$$

No Answer!

(14)

$$\begin{aligned}5^{\sqrt{x+1}} &= 1000 \\(\sqrt{x+1})\ln(5) &= \ln(1000) \\\sqrt{x+1} &= 4.292 \\\sqrt{x} &= 3.292 \\x &= 10.837\end{aligned}$$

(15)

$$\begin{aligned}\ln(X-3) &= 2 - \ln(X-4) \\\ln(X-3) + \ln(X-4) &= 2 \\\ln((X-3)(X-4)) &= 2 \\X^2 - 7X + 12 &= e^2 \\X^2 - 7X + 4.611 &= 0 \\&= \frac{-(-7) \pm \sqrt{49 - 4(1)(4.611)}}{2} \\&= \frac{7 \pm \sqrt{30.556}}{2} \\X &= 6.264 \\&\text{or} \\X &= .7361\end{aligned}$$

(16a)

$$\begin{aligned}\left(9a^{\frac{2}{5}}b^{-4}\right)^{-\frac{1}{2}} \\3a^{-\frac{1}{5}}b^2 &= \frac{3b^2}{a^{\frac{1}{5}}}\end{aligned}$$

(16b)

$$\sqrt[5]{100.00} = 2.5119$$

(16c)

$$\begin{aligned}\left(-64a^{-12}\right)^{\frac{1}{3}} \\-4a^{-4} &= \frac{-4}{a^4}\end{aligned}$$

(16d)

$$1000e^{-2} = 818.731$$

(16e)

$$a\sqrt{a} = a^1 a^{1/2} = a^{3/2}$$

(16f)

$$\begin{aligned}(2 \times 10^3)^3 &= (2000)^3 \\ &= 8 \times 10^9\end{aligned}$$

(17)

Betty:

$$4000(1+R)^{27/12} = 5,478.10$$

$$(1+R)^{2.25} = 1.369525$$

$$1+R = 1.1499998$$

$$R = 15\%$$

Bob:

$$16,000(1+R)^{44/12} = 25,868.49$$

$$(1+R)^{3.6667} = 1.616781$$

$$1+R = 1.1399999$$

$$R = 14\%$$

So, Betty did better!

(18a)

$$2P = A = Pe^{rt}$$

$$2 = e^{rt}$$

$$\ln(2) = rt$$

$$\frac{\ln(2)}{r} = t$$

(18b)

$$5000e^{6.93\% \cdot 40} = \$79,952.92$$

(19a)

$$25000(1+.20)^T = 12,500(1+.40)^T$$

$$2(1.2)^T = (1.4)^T$$

$$\ln(2) + T \ln(1.2) = T \ln(1.4)$$

$$.6931 + .18232T = .33647T$$

$$.6931 = .15415T$$

$$T = 4.4963$$

(19b)

$$\begin{aligned}2(1+R)^T &= (1+2R)^T \\ \ln(2) + T \ln(1+R) &= T \ln(1+2R) \\ \ln(2) &= [\ln(1+2R) - \ln(1+R)]T \\ T &= \frac{\ln(2)}{\ln\left(\frac{1+2R}{1+R}\right)}\end{aligned}$$

(20a)

Yes. Good for all r 's.

(20b)

No! $\log\left(\frac{A}{B}\right) = \log\left(\frac{C}{D}\right)$

(20c)

$$\frac{e^{x^2}}{e^x} = e^{x^2} e^{-x} = e^{(x^2-x)}$$

(20d)

$$S = V^{2/3} \times 6$$

(20e)

$$\log(\sqrt[n]{a}) = \log\left(a^{1/n}\right) = \frac{1}{n} \log(a)$$