Solutions To Project 2

(1)

$$\begin{aligned} 4x - 7 &= 3(8 - 2x) \\ 4x - 7 &= 24 - 6x \\ 10x &= 31 \\ x &= \frac{31}{10} \\ &= 3.1 \end{aligned}$$

(2)

$$\begin{aligned} \frac{1}{2}(x - 6) &= \frac{3}{5}x + 1 \\ \frac{1}{2}x - 3 &= \frac{3}{5}x + 1 \\ -\frac{1}{10}x &= 4 \\ x &= -10(4) \\ &= -40 \end{aligned}$$

(3)

$$\begin{aligned} \frac{1}{2}\left[1 + \frac{1}{4}(3x - 1)\right] &= \frac{2x}{3} - \frac{1}{2} \\ 24\left[\frac{1}{2} + \frac{1}{8}(3x - 1)\right] &= 24\left[\frac{2x}{3} - \frac{1}{2}\right] \\ 12 + 3(3x - 1) &= 16x - 12 \\ 12 + 9x - 3 &= 16x - 12 \\ 21 &= 7x \\ x &= 3 \end{aligned}$$

(4)

$$\begin{aligned} 3[2x + 1 - 2(2x - 1)] + 4 &= 2[1 + 2(3 - x)] \\ 3[2x + 1 - 4x + 2] + 4 &= 2[1 + 6 - 2x] \\ 3[-2x + 3] + 4 &= 2[7 - 2x] \\ -6x + 9 + 4 &= 14 - 4x \\ -6x + 13 &= 14 - 4x \\ -1 &= 2x \\ x &= -\frac{1}{2} \end{aligned}$$

(5)

$$\frac{2x-3}{4x-5}=\frac{2}{5}$$
$$(2x-3)5=2(4x-5)$$
$$10x-15=8x-10$$
$$2x=5$$
$$x=\frac{5}{2}=2.5$$

(6)

$$\frac{2x}{x-3}-\frac{5}{x}=2$$
$$2x^2-5(x-3)=2(x-3)$$
$$2x^2-5x+15=2x^2-6$$
$$x=-15$$

(7)

$$\frac{4}{x}-\frac{3}{x^2}=\frac{5}{2x}$$
$$4x-3=\frac{5}{2}x; x\neq 0$$
$$\frac{3}{2}x=3$$
$$x=\frac{2}{3}(3)=2$$

(8)

$$2x^2-7x+3=0$$
$$(2x-1)(x-3)=0$$
$$x=\frac{1}{2}, x=3$$

(9)

$$5x+2=3x^2$$
$$0=3x^2-5x-2$$
$$0=(3x+1)(x-2)$$
$$x=-\frac{1}{3}, x=2$$

(10)

$$\frac{1}{F}=\frac{1}{A}+\frac{1}{B}$$
$$\frac{1}{F}-\frac{1}{B}=\frac{1}{A}$$
$$\frac{B-F}{FB}=\frac{1}{A}$$
$$A=\frac{BF}{B-F}$$

(11)

$$S=\frac{A-RL}{1-R}$$
$$S-SR=A-RL$$
$$S-A=SR-RL=R(S-L)$$
$$R=\frac{S-A}{S-L}$$

(12)

$$A=P+PRT$$
$$A=P(1+RT)$$
$$P=\frac{A}{1+RT}$$

(13)

$x^4+x^2-12=0$

Let $u=x^2$ so $u^2=x^4$

and $u^2+u-12=0$

$(u-3)(u+4)=0$

$u=3$ or $u=-4$

$x^2=3$ or $x^2=-4$

$x=\sqrt{3}$ or $-\sqrt{3}$

(14)

$$(x-1)(x+3)=(x+2)(x-3)+1$$
$$x^2-x+3x-3=x^2-3x+2x-6+1$$
$$2x-3=-x-5$$
$$3x=-2$$
$$x=-\frac{2}{3}$$

(15)

$$3x(2x-5) = -4x-3$$

$$6x^2 - 15x = -4x - 3$$

$$6x^2 - 11x + 3 = 0$$

$$(2x-3)(3x-1) = 0$$

$$x = \frac{3}{2}, x = \frac{1}{3}$$

(16)

$$A = 2\prod R(R+H)$$

$$0 = 2\prod R^2 + 2\prod RH - A$$

$$R = \frac{-2\prod H \pm \sqrt{4\prod^2 H^2 - 4(2\prod)(-A)}}{4\prod}$$

$$R = \frac{-2\prod H \pm \sqrt{4\prod^2 H^2 + 8\prod A}}{4\prod}$$

$$R = \frac{-2\prod H \pm 2\sqrt{\prod^2 H^2 + 2\prod A}}{4\prod}$$

$$R = \frac{-\prod H \pm \sqrt{\prod^2 H^2 + 2\prod A}}{2\prod}$$

or $$R = \frac{-H \pm \sqrt{H^2 + \frac{2A}{\prod}}}{2}$$

(17)

$$(2x+1)^2 = 3(x+1)^2$$

$$4x^2 + 4x + 1 = 3(x^2 + 2x + 1)$$

$$4x^2 + 4x + 1 = 3x^2 + 6x + 3$$

$$x^2 - 2x - 2 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2}$$

$$x = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

$$x = 1 \pm \sqrt{3}$$

$$= 2.732, -.732$$

Note: $A^2 = B^2 \Leftrightarrow A = \pm B$

So Solve $2x+1=\sqrt{3}(x+1)$

or $2x+1=-\sqrt{3}(x+1)$

and get same results as above.

(18)

$4x^2 = 5x$

1. $x = 0$ is a solution.
2. if $x \neq 0$

$$\frac{4x^2}{x} = \frac{5x}{x}$$

$$4x = 5$$

$$x = \frac{5}{4}$$

The solutions are 0 and $\frac{5}{4}$

(19)

$$D = \frac{360}{T}\left(\frac{100-P}{100}\right)$$

$$\frac{DT(100)}{360} = 100 - P$$

$$P = 100 - \frac{100DT}{360}$$

(20)

$$x^2 = 3 - 2x$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1, -3$$

(21)

Let x = money she came in with.

$$\frac{5}{9}x + \frac{1}{6}x + \frac{1}{12}x + 26.25 = x$$

$$36\left(\frac{5}{9}x + \frac{1}{6}x + \frac{1}{12}x + 26.25\right) = 36x$$

$$20x + 6x + 3x + 945 = 36x$$

$$945 = 36x - 29x$$

$$945 = 7x$$

$$x = \frac{945}{7} = 135$$

(22)

$$\text{Let } x = \text{Betty's bill} \Rightarrow 48 - x \text{ is Bob's bill}$$

$$\text{Now } .15x + .25(48 - x) = 10.20$$

$$\text{So } .15x + 12 - .25x = 10.20$$

$$1.80 = .1x$$

$$x = 18$$

So Betty's meal cost \$18; tip \$2.5
Bob's meal cost \$30; tip 7.5

(23)

$$103.7721 = \frac{6}{1+R} + \frac{106}{(1+R)^2}; 0 < R$$

$$\text{Let } x = \frac{1}{1+R} \text{ whence } 0 < x \text{ and}$$

$$103.7721 = 6x + 106x^2$$

$$0 = 106x^2 + 6x - 103.7721$$

$$x = \frac{-6 \pm \sqrt{36 - 4(106)(-103.7721)}}{212}$$

$$x = \frac{-6 \pm 209.8461}{212} = .9615, -1.018$$

$$\text{Now } \frac{1}{x} = 1 + R \Rightarrow R = \frac{1}{x} - 1$$

$$\text{So } R = \frac{1}{.9615} - 1 = .04$$

(24)

$$A = P(1 + RT)$$

$$\frac{A}{P} = 1 + RT$$

$$\frac{A}{P} - 1 = RT \Rightarrow R = \frac{A/P - 1}{T}$$

$$\text{Tom: } R = \frac{\frac{12{,}000}{10{,}000} - 1}{.75} = .267$$

$$\text{Nicole: } R = \frac{\frac{11{,}500}{8{,}000} - 1}{1.333} = .328$$

Nicole did better. Her rate of return is greater than Tom's.

(25a)

Rule should read $\frac{A}{B} = 0$ if and only if $A = 0, B \neq 0$

(25b)

$\sqrt{\frac{1}{A}} = \frac{\sqrt{1}}{\sqrt{A}} = \frac{1}{\sqrt{A}}; 0 < A$ Valid rule.

(25c)

$X + X = 2X$ Every number is a solution.

(25d)

$(X - 1)^2 = 0$ has one solution, $X = 1$.

Clearly $(X - d)^2 = 0$ has one solution, $X = d$

(25e)

$(x-1)(x-2)(x-3) = 0$

$x = 1,2,3$ are the solutions.

(25f)

$\frac{2}{x} = 0$ has no solutions.