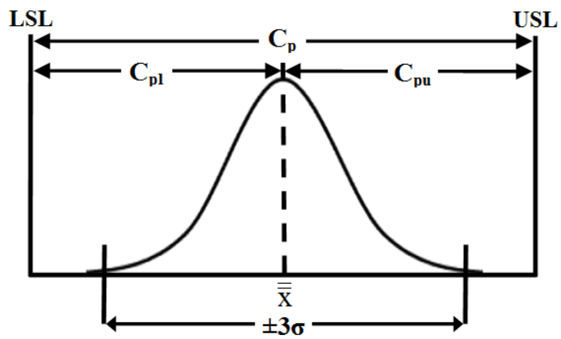
Lesson 9
Statistics

The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

The area under a normal curve from -∞ to +∞ is 1. This is because a normal distribution is a probability density function. The density is symmetric about μ. The points of inflection occur at x = μ ± σ.



Recall that the standard normal distribution is:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

Because no normal distribution is integratable, we use the standard normal distribution *tables* to evaluate:

$$F(z) = p(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

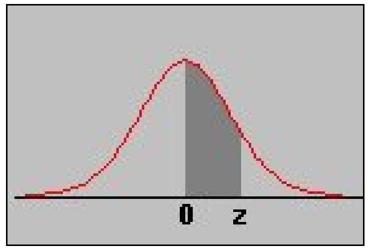
We then scale the statistic from the standard normal to a given normal distribution via:

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \Phi(z)$$

Where:

$$x = \mu + z\sigma$$

To find the area under a normal curve we use the standard normal z-tables. Here is a portion of that table:



	0.00	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	**0.1293**	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	**0.2486**	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621

Ex. 1:

What is the probability that a random number drawn from a distribution that is N~(10, 9) falls between 8 and 9?

So, we must find the probability of a random number being less than 9 and subtract from that the probability of the number being less than 8. Thus,

$$P(8 \le x \le 9) = \int_{8}^{9} \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-10}{3}\right)^2} dx = F(9) - F(8)$$

Find the corresponding z’s in the standard normal distribution:

$$z_8 = \frac{8-10}{3} = \frac{-2}{3}$$
$$z_9 = \frac{9-10}{3} = \frac{-1}{3}$$

Now the tables only show z values greater than 1, but since the standard normal is symmetric, we can use the absolute value. Now,

$$P(-\infty < z < .33) = .5 +.1293 = .6293$$

And,

$$P(-\infty < z < .67) = .5 + .2486 = .7486$$

However, this is not the area we want. What we want is:

$$P(.33 < z < \infty) = 1 - .6293 = .3707$$
$$P(.67 < z < \infty) = 1 - .7486 = .2514$$

Because the standard normal is symmetric:

$$P(-.67 < z < -.33) = .3707 - .2514 = .1193$$

Project 9

Find the area under the curve:

1. $f(x) = 2e^{x} \quad 0 \le x \le 2$

2. $f(x) = .25x^{2} \quad 1 \le x \le 3$

3. $f(x) = 5 \quad 4 \le x \le 10$

Find the value of *k* that makes each function a density:

4. $f(x) = k \cdot \frac{1}{2} x^{2} \quad 0 \le x \le 2$

5. $f(x) = ke^{2x} \quad 0 \le x \le 3$

6. $f(x) = k(2x+2)^{2} \quad 0 \le x \le 1$

Find the expected value and variance of each function.

7. $f(x) = k \cdot 3x^{1/2} \quad 0 \le x \le 4$

8. $f(x) = k \cdot e^{1/2x} \quad 0 \le x \le 2$

9. $f(x) = 4k \quad 0 \le x \le 3$